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TECHNICAL MEMORANDUM 1258

FLIGHT PERFORMANCE OF A JET POWER PLANT

III - OPERATING CHARACTERISTICS OF A JET POWER PLANT AS A FUNCTION OF ALTITUDE

By F. Weinig

Translation

"Flugmechanik des Strahltre ibwerks. III - Betriebsverhalten eines Strahltreibwerks in Abhängigkeit von der Flughöhe." FB 1743/3, 2WB, Nov. 1, 1943. (Forschungsinstitut f. Kraftfahrwesen u. Fahrzeugmotoren, Tech. Hoch. (Stuttgart).)

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III - OPERATING CHARACTERISTICS OF A JET POWER PLANT AS A FUNCTION OF ALTITUDE*

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SUMMARY

The performance of a jet power plant consisting of a compressor and a turbine is determined by the characteristic curves of these component parts and is controllable by the characteristics of the compressor and the turbine in relation to each other. The normal output, overload, and throttled load of the jet power plant are obtained on the basis of assumed straight-line characteristics.

1. NOTATION

(See fig. 1.)

$J = c_p T$	energy head, (m)
S	entropy, $(m/^{\circ})$
T	temperature, $^{\circ}K$
p	pressure, (kg/m^2)
v	specific volume, (cu m/kg)
$\gamma = 1/w$	specific weight, (kg/cu m)
$H_B = H_u/A$	heat drop of fuel, (m)
H_L	theoretical delivery head of compressor, (m)

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H_T	theoretical drop of turbine, (m)
G_B	weight of fuel supplied per unit time, (kg/sec)
G_L	weight of air supplied per unit time, (kg/sec)
ΔT_B	temperature rise in combustion chamber, (°)
$R = c_p - c_v$	gas constant, (m/°)
c_p	specific heat at constant pressure, (m/°)
c_v	specific heat at constant volume, (m/°)
u	peripheral velocity, (m/sec)
v	axial velocity, (m/sec)
v_o	flight speed, (m/sec)
h	flight altitude, (km)
w_s	velocity of sound, (m/sec)
$N_B = G_B H_B$	power contained in fuel, (mkg/sec)
$N_e = \eta_e N_B$	jet power output, (mkg/sec)
$N_w = \eta_w N_B = v_o$	propulsive output, (mkg/sec)
S	thrust in flight, (kg)
S_O	thrust at standstill, (kg)

Subscripts:

0	in atmosphere or at rest
1 a	compressor inlet
b	compressor outlet and combustion-chamber inlet
c	combustion-chamber outlet and turbine inlet
2 d	turbine outlet and nozzle inlet

3	nozzle exit
3_1	nozzle outlet under standstill conditions
L	compressor
T	turbine
N	layout state

Characteristic magnitudes:

$\vartheta = \vartheta_1 = \frac{T_1}{\Delta T_B}$ or $\vartheta_0 = \frac{T_0}{\Delta T_B}$ measure of temperature rise in combustion chamber

$\pi = \frac{T_b}{T_a} = \left(\frac{p_b}{b_a}\right)^{\frac{k-1}{\eta_L k}}$ temperature ratio in compressor

$\Lambda = \frac{T_b - T_a}{\Delta T_B} = (\pi - 1)\vartheta$ measure for loading of compressor

$\theta = T_c/T_a = \pi + 1/\vartheta$ measure for heat stress of turbine

$M = \frac{v}{W_{S0}} = \frac{v}{\sqrt{gkRT_0}}$ Mach number of flight condition

$M_L = \frac{u_L}{W_{S0}} = \frac{u_L}{\sqrt{gKRT_1}}$ Mach number of compressor

$\psi = \frac{2gH}{u^2}$ pressure coefficient

$\varphi = \frac{v}{u}$ flow coefficient

$k = -d\frac{\psi}{\psi_N}\Big/ d\frac{\varphi}{\varphi_N}$ steepness of characteristic curves near layout condition

$h' = k_L \varphi_{L_N}$ characteristic parameter of compressor [Ed. note: Symbol illegible in German document; assumed to be h'.]

$j = \frac{K_L \varphi_{L_N}}{k_T \varphi_{T_N}}$ characteristic parameter for relative design of compressor and turbine

$d = \frac{j-1}{h'(1-1/\pi_N)} = \frac{1}{1-\pi_N}\left(\frac{1}{k_T \varphi_{T_N}} - \frac{1}{k_L \varphi_{L_N}}\right)$ design parameter

$g = \frac{G_B/\gamma_a}{G_{B_N}/\gamma_{a_N}}$ ratio for fuel consumption at any operating condition

$t_0 = \frac{\eta_{eo}}{\eta_{ea}}$ ratio of power plant efficiency for any operating condition

$f_0 = f_0 g$ output ratio for any operating condition

$k = c_p/c_v$ adiabatic exponent

m_L polytropic exponent in compressor

m_T polytropic exponent in turbine

η_L efficiency of flow of compressor

η_{iaDL} internal adiabatic efficiency of compressor

η_{mL} mechanical efficiency of compressor

η_T flow efficiency of turbine

η_{iaiT} internal adiabatic efficiency of turbine

η_{m_T} mechanical efficiency of turbine

η_m	over-all mechanical efficiency of jet engine = $\eta_{m_L} \eta_{m_T} \left(1 + \frac{G_B}{G_L}\right)$
η_g	efficiency measure
η_v	efficiency of complete machine
η_i	internal efficiency
η_d	nozzle efficiency
η_{e_0}	power-plant efficiency at standstill
$\Delta\eta_e$	increment in flight
$\eta_e = \eta_{e_0} + \Delta\eta_e$	power-plant efficiency
η_p	propulsive efficiency measure
η_w	specific power output
σ	[Ed. note: German text illegible] thrust efficiency
σ_0	[Ed. note: German text illegible] standstill thrust efficiency

2. PERFORMANCE OF JET POWER PLANT WITH ROTATIONAL SPEED REGULATION

A jet engine consisting of compressor and turbine can be regulated with the aid of the fuel supply and a suitable outlet nozzle with respect to the rotational speed and the amount of air flow and therefore with respect to the turbine-inlet temperature. From the rotational speed and the amount of air flow, the compressor pressure ratio and therefore the compressor temperature ratio $\pi = T_b/T_a$ are determined. Through the turbine-inlet temperature or the temperature rise in the combustion chamber, there is then also determined the temperature-rise ratio of the combustion chamber, namely, $\frac{1}{\delta} = \frac{c_{p_B}\,\Delta T_B}{c_{p_L}\,T_a}$.

For characterizing the operating condition of the jet engine, the characteristic parameters can therefore be employed:

$$\pi = \frac{T_b}{T_a} \tag{2.1a}$$

$$\vartheta = \frac{c_{p_L} T_a}{c_{p_B} \Delta T_B} \tag{2.1b}$$

By one of these two parameters, the internal mechanical similarity of a given jet power plant for various operating conditions is characterized, at least as long as their mutual relations are not influenced through other regulating means than through the regulation of the rotational speed with the aid of the fuel supply for a correspondingly suitable outlet nozzle. For such a jet engine regulated only by the fuel and outlet nozzle, there is to be determined from a test stand the functional relation

$$\vartheta = \vartheta(\pi)$$

[Ed. note: $\vartheta = \vartheta(\pi) = - \frac{c_{p_L}}{c_{p_B}}\left(\frac{T_b - T_a}{\Delta T_B}\right)$.]

from which corresponding values of the parameters ϑ and π are obtained. In place of these parameters there may also be introduced

$$\Lambda = \frac{T_b - T_a}{\Delta T_B} = \frac{t_b - t_a}{\Delta t_B} \tag{2.1c}$$

$$\theta = \frac{T_c}{T_a} \tag{2.1d}$$

As a relation between these parameters there is then obtained

$$\Lambda = (\pi - 1)\vartheta \tag{2.1e}$$

and

$$\theta = \pi + \frac{1}{\vartheta} \tag{2.1f}$$

[Ed. note: Equations (2.1c) and (2.1f) are based on $c_{p_L} = c_{p_B}$.]

Two magnitudes limit the output of a jet engine. One is the temperature of the combustion chamber T_c at the inlet to the turbine, which must be withstood by the turbine blades. The other is the Mach number for the inlet air into the compressor. For the Mach number there may be used

$$M_L = \frac{u}{w_{s_1}} \tag{2.2}$$

that is, the ratio of the peripheral speed in the first stage of the compressor to the sound velocity corresponding to the inlet temperature T_1

$$w_{s_1} = \sqrt{gkRT_1} \tag{2.2a}$$

The exceeding of a certain Mach number means a sharp drop in the efficiency.

Under the operating condition of the jet engine, there must be obtained for the work drop H_T of the turbine and the output pressure head H_L of the compressor

$$H_T = (J_c - J_d) = \frac{1}{\eta_m}(J_b - J_a) = \frac{1}{\eta_m} H_L \tag{2.3}$$

For the operation of the turbine at constant inlet temperature, if its characteristic curve at the layout condition N is assumed as linear (fig. 2), there is obtained in nondimensional form

$$\frac{2gH_T}{u_T^2} = \left(\frac{2gH_T}{u_T^2}\right)_Z \cdot \left\{1 - k_T\left[\frac{v_T}{u_T} - \left(\frac{v_T}{u_T}\right)_N\right]\right\} \tag{2.4}$$

and for constant Mach number at the compressor inlet if its characteristic curve may be assumed in the neighborhood of the layout state as linear

$$\frac{2gH_L}{u_L^2} = \left(\frac{2gH_L}{u_L^2}\right)_Z \cdot \left\{1 - k_L\left[\frac{v_L}{u_L} - \left(\frac{v_L}{u_L}\right)_N\right]\right\} \tag{2.5}$$

where

v axial component of velocity entering compressor or turbine

u peripheral velocity

Subscript T refers to the turbine; subscript L, to the compressor.

For briefness, for turbine at design point

$$\left(\frac{2gH_T}{u_T^2}\right)_N = \psi_{T_N}$$

$$\left(\frac{v_T}{u_T}\right)_N = \varphi_{T_N}$$

for turbine at off design

$$\frac{2gH_T}{u_T^2} = \psi_T$$

$$\frac{v_T}{u_T} = \varphi_T$$

for compressor at design point

$$\left(\frac{2gH_L}{u_L^2}\right)_N = \psi_{L_N}$$

$$\left(\frac{v_L}{u_L}\right)_N = \varphi_{L_N}$$

for compressor at off design.

$$\frac{2gH_L}{u_L^2} = \psi_L$$

$$\frac{v_L}{u_L} = \varphi_L$$

where ψ is the pressure coefficient and φ the flow coefficient. The subscript N refers to the layout condition. Then

$$\frac{u_T}{u_L} = i \tag{2.6}$$

Then

$$i^2\eta_m\left(\frac{2gH_T}{u_T^2}\right)_N = \left(\frac{2gH_L}{u_L^2}\right)_N \tag{2.7}$$

$$i^2\eta_m \frac{2gH_T}{u_T^2} = \frac{2gH_L}{u_L^2} \tag{2.8}$$

Therefore

$$k_T\left(\frac{v_T}{u_T} - \varphi_{T_N}\right) = k_L\left(\frac{v_L}{u_L} - \varphi_{L_N}\right) \tag{2.9}$$

In the design state, v shall denote the specific volume at the inlet to the compressor and turbine

$$v_T = (v_T)_N$$

$$v_L = (v_L)_N$$

$$\left(\frac{v_T}{v_L}\right)_N = m\left(\frac{v_T}{v_L}\right)_N \tag{2.10}$$

Then

$$\frac{\varphi_{T_N}}{\varphi_{L_N}} = \frac{(v_T)_N}{(v_L)_N} \frac{u_L}{u_T} = \frac{m}{i}\left(\frac{v_T}{v_L}\right)_N \tag{2.11}$$

Hence if $\left(\frac{v_T}{v_L}\right)\left(\frac{u_L}{u_T}\right) = \left(\frac{m}{i}\right)\left(\frac{V_T}{V_L}\right)$

$$\frac{m}{i} k_T \left[\frac{v_L}{u_L}\frac{V_T}{u_L} - \varphi_{L_N}\left(\frac{V_T}{V_L}\right)_N\right] - k_L\left(\frac{v_L}{u_L} - \varphi_{L_N}\right)$$

or

$$\frac{v_L}{u_L}\left(\frac{m}{i}\frac{k_T}{k_L}\frac{V_T}{V_L} - 1\right) = \varphi_{L_N}\left[\frac{m}{i}\frac{k_T}{k_L}\left(\frac{V_T}{V_L}\right)_N - 1\right] \tag{2.12}$$

and if

$$\frac{\varphi_{L_N}}{\varphi_{T_N}}.\frac{k_L}{k_T} = \left(\frac{V_L}{V_T}\right)\frac{ik_L}{mk_T} = j \tag{2.12a}$$

$$k_L\,\varphi_{L_N} = h' \tag{2.12b}$$

Then

$$\psi_L = \frac{2gH_L}{{u_L}^2} = \psi_{L_N}\left[1 - h'\,\frac{\left(\frac{V_T}{V_L}\right)_N - \frac{V_T}{V_L}}{\frac{V_T}{V_L} - j\left(\frac{V_T}{V_L}\right)_N}\right] \tag{2.13}$$

When

$$H_L = c_p\,(T_b - T_a)$$

$$\frac{T_c}{T_a} = \frac{1 + \vartheta\pi}{\vartheta} = \theta$$

$$u^2 = M_L{}^2\, w_{s_1}{}^2 = M_L{}^2\, gkRT_a = M_L{}^2\, gk(c_p - c_v)\, T_a = M_L{}^2\, gc_p\, (k-1)\, T_a$$

with $p_b = p_c$, and when

$$\left(\frac{V_T}{V_L}\right)_N = \left(\frac{T_c}{T_a}\right)_N \left(\frac{p_a}{p_b}\right)_N = \frac{1 + \vartheta_N\, \pi_N}{\vartheta_N} \frac{1}{\pi_N{}^{\frac{\eta_L k}{k-1}}} = \frac{\theta_N}{\pi_N{}^{\frac{\eta_L k}{k-1}}} \tag{2.14}$$

$$\frac{V_T}{V_L} = \frac{T_c}{T_a}\frac{p_a}{p_b} = \frac{1 + \vartheta\pi}{\vartheta} \frac{1}{\pi^{\frac{\eta_L k}{k-1}}} = \frac{\theta}{\pi^{\frac{\eta_L k}{k-1}}} \tag{2.15}$$

Thus

$$\frac{T_b - T_a}{T_a} = \pi - 1 = \frac{k-1}{2} M_L{}^2\, \psi_{L_N} \left(1 - h' \frac{\dfrac{\theta_N}{\pi_N{}^{\frac{\eta_c k}{k-1}}} - \dfrac{\theta}{\pi^{\frac{\eta_L k}{k-1}}}}{\dfrac{\theta}{\pi^{\frac{\eta_L \theta}{k-1}}} - j \dfrac{\theta_N}{\pi_N{}^{\frac{\eta_L k}{k-1}}}}\right). \tag{2.16}$$

For the layout condition,

$$\left(\frac{T_b - T_a}{T_a}\right)_N = \pi_N - 1 = \frac{k-1}{2} M_{L_N}{}^2\, \psi_{L_N} \tag{2.17}$$

[Ed. note: The M term raised to second power by reviewer.]

so that, in general,

$$1 - h'\left(\frac{\dfrac{\theta_N}{\pi_N^{\frac{\eta_L k}{k-1}}} - \dfrac{\theta}{\pi^{\frac{\eta_L k}{k-1}}}}{\dfrac{\theta}{\pi^{\frac{\eta_L k}{k-1}}} - j\dfrac{\theta_N}{\pi^{\frac{\eta_L k}{k-1}}}}\right) = \left(\frac{M_{L_N}}{M_L}\right)^2 \frac{\pi-1}{\pi-1}$$

The regulation is to be such that the Mach number M_L is constant so that

$$\frac{M_L}{M_{L_N}} = 1$$

Hence

$$h' \frac{\dfrac{\theta_N}{\pi_N^{\frac{\eta_L K}{k-1}}} - \dfrac{\theta}{\pi^{\frac{\eta_L K}{k-1}}}}{\dfrac{\theta}{\pi^{\frac{\eta_L K}{K-1}}} - j\dfrac{\theta_N}{\pi_N^{\frac{\eta_L K}{K-1}}}} = 1 - \frac{\pi-1}{\pi_N-1} = \frac{\pi_N-\pi}{\pi_N-1}$$

[Ed. note: In the j term, the subscript N was omitted from θ.]

and solving for θ/θ_N

$$\frac{\theta}{\theta_N} = \left(\frac{\pi}{\pi_N}\right)^{\frac{\eta_L K}{K-1}} \frac{1 - \dfrac{j}{h'\left(1 - \dfrac{1}{\pi_N}\right)}\left(\dfrac{\pi}{\pi_N} - 1\right)}{1 - \dfrac{1}{h'\left(1 - \dfrac{1}{\pi_N}\right)}\left(\dfrac{\pi}{\pi_N} - 1\right)} \tag{2.18}$$

By differentiating, there is obtained

$$\frac{d\,\frac{\theta}{\theta_N}}{d\,\frac{\pi}{\pi_N}} = \frac{\theta}{\theta_N}\left[\frac{\frac{\eta_L K}{k-1}}{\frac{\pi}{\pi_N}} + \frac{\frac{1}{h'\left(1 - \frac{1}{\pi_N}\right)}}{1 - \frac{1}{h'\left(1 - \frac{\pi}{\pi_N}\right)}\left(\frac{\pi}{\pi_N} - 1\right)} - \frac{\frac{j}{h'\left(1 - \frac{1}{\pi_N}\right)}}{1 - \frac{j}{h'\left(1 - \frac{1}{\pi_N}\right)}\left(\frac{\pi}{\pi_N} - 1\right)}\right]$$

In the neighborhood of the layout condition $(\theta = \theta_N, \quad \pi = \pi_N)$,

$$\left(\frac{d\,\frac{\theta}{\theta_N}}{d\,\frac{\pi}{\pi_N}}\right)_N = \frac{\eta_L k}{k-1} - \frac{j-1}{h'\left(1 - \frac{1}{\pi_N}\right)}$$

As a first approximation, for the neighborhood of the layout condition there may be written

$$\pi = \pi_N\left(\frac{\theta}{\theta_N}\right)^{\dfrac{1}{\frac{\eta_L k}{k-1} - \frac{j-1}{h'\left(1 - \frac{1}{\pi_N}\right)}}} \tag{2.19}$$

(fig. 3)

Correspondingly, there is

$$\vartheta = \frac{1}{\theta - \pi} \qquad \frac{1}{\dfrac{1}{\dfrac{\eta_L K}{K-1} - \dfrac{j-1}{h'\left(1 - \dfrac{1}{\pi_N}\right)}}} \tag{2.20}$$

(fig. 4)

$$\theta_N\left(\frac{\theta}{\theta_N}\right) - \pi_N\left(\frac{\theta}{\theta_N}\right)$$

In general, from equation (2.19)

$$\pi = \pi_N(\theta) \tag{2.19a}$$

[Ed. note: Subscript N supplied by reviewer.]

and

$$\vartheta = \frac{1}{\theta - \pi_N(\theta)} = \vartheta(\theta) \tag{2.20a}$$

[Ed. note: Subscript N supplied by reviewer.]

Under the assumption of straight characteristic curves (equations (2.4) and (2.5)), the values will hold also for greater deviations from the assumed design conditions.

This relation can then be suitably represented graphically. By substituting the relations found for π and θ in the equation for η_g and η_v there is obtained

$$\eta_g(\theta) = 1 - (\pi(\theta) - 1)\left(\frac{1}{\eta_m} - 1\right)\vartheta(\theta) \tag{2.21}$$

and

$$\eta_i(\theta) = \left[1 + \vartheta(\theta)\,\frac{1 - \left(1-\eta_m\right)\pi(\theta)}{\eta_m}\right] \cdot \left\{1 - \frac{1}{\pi(\theta)^{\eta_L}}\left[\frac{1 + \vartheta(\theta)\,\pi(\theta)}{1 + \vartheta(\theta)\,\frac{1 - \left(1-\eta_m\right)\pi(\theta)}{\eta_m}}\right]^{\frac{1}{\eta_T}}\right\} \tag{2.22}$$

There is also obtained

$$\eta_v(\theta) = \frac{\eta_i}{\eta_g} = \frac{\left[1+\vartheta(\theta)\,\frac{1-(1-\eta_m)\pi(\theta)}{\eta_m}\right]\left\{1 - \frac{1}{\pi(\theta)^{\eta_L}}\left[\frac{1+\vartheta(\theta)\pi(\theta)}{1+\vartheta(\theta)\,\frac{1-(1-\eta_m)\pi(\theta)}{\eta_m}}\right]^{\frac{1}{\eta_T}}\right\}}{1-(\pi(\theta)-1)\left(\frac{1}{\eta_m} - 1\right)\vartheta(\theta)} \tag{2.23}$$

and

$$\eta_{e_0}(\theta) = \eta_d\,\eta_i = \eta_d\left[1 + \vartheta(\theta)\,\frac{1-\left(1-\eta_m\right)\pi(\theta)}{\eta_m}\right]\left\{1 - \frac{1}{\pi(\theta)^{\eta_L}}\left[\frac{1 + \vartheta(\theta)\,\pi(\theta)}{1+\vartheta(\theta)\,\frac{1-(1-\eta_m)\pi(\theta)}{\eta_m}}\right]^{\frac{1}{\eta_T}}\right\} \tag{2.24}$$

Thus a relation has been established, which for suitable adjustment of the outlet nozzle makes possible the computation of the efficiency of a jet engine in the neighborhood of the design condition if the fuel distribution is such that the Mach number at the inlet to the compressor M_L and the inlet temperature T_0 into the turbine are held constant. If this Mach number M_L and inlet temperature T_0 are the highest permissible values for the jet engine, there is obtained the engine efficiency as a function of the inlet temperature for the admissible maximum output

$$T_a = T_1 = \frac{T_c}{\theta}$$

This maximum output is obtained from

$$N_{e_o} = \eta_{e_o} H_B G_B = \eta_{e_o} c_p \Delta T_B (G_L + G_B)$$

whence

$$\frac{N_{e_o}}{N_{e_{o_N}}} = \left(\frac{\eta_{e_o}}{\eta_{e_{o_N}}}\right)\left(\frac{\Delta T_B}{\Delta T_{B_N}}\right)\left(\frac{G_L + G_B}{G_{L_N} + G_{B_N}}\right)$$

$$\frac{N_{e_o}}{N_{e_{o_N}}} = \left(\frac{\eta_{e_o}}{\eta_{e_{o_N}}}\right)\left(\frac{\Delta T_B}{\Delta T_{B_N}}\right)\left(\frac{G_L}{G_{L_N}}\right)\left(\frac{1 + \frac{G_B}{G_L}}{1 + \frac{G_{B_N}}{G_{L_N}}}\right) \tag{2.25}$$

where

$$\frac{G_L}{G_{L_N}} = \frac{v_L/V_L}{v_{L_N}/V_{L_N}} = \left(\frac{v_L/u_L}{v_{L_N}/u_{L_N}}\right)\left(\frac{u_L}{u_{L_N}}\right)\left(\frac{V_{L_N}}{V}\right)$$

or with $M_L = M_{L_N}$

$$\frac{U_L}{U_{L_N}} = \left(\frac{M_L}{M_{L_N}}\right)\left(\frac{w_s}{w_{s_N}}\right) = \sqrt{\frac{T_a}{T_{a_N}}}$$

$$\frac{V_{L_N}}{V_L} = \left(\frac{T_{a_N}}{T_a}\right)\left(\frac{p_a}{p_{a_N}}\right)$$

From equations (2.12), (2.14), and (2.15),

$$\frac{\dfrac{v_L}{u_L}}{\dfrac{v_{L_N}}{u_{L_N}}} = \frac{\dfrac{1}{j} - 1}{\dfrac{1}{j}\left[\dfrac{\dfrac{V_T}{V_L}}{\left(\dfrac{V_T}{V_L}\right)_N}\right] - 1} = \frac{j - 1}{j - \dfrac{\theta}{\theta_N}\left(\dfrac{\pi_N}{\pi}\right)^{\frac{\pi_L k}{k-1}}}$$

$$\frac{G_L}{G_{L_N}} = \frac{j - 1}{j - \dfrac{\theta}{\theta_N}\left(\dfrac{\pi_N}{\pi}\right)^{\frac{\eta_L k}{k-1}}}\sqrt{\frac{T_{a_N}}{T_a}\,\frac{p_a}{p_{a_N}}} \tag{2.26}$$

Further

$$\frac{\Delta T_B}{\Delta T_{B_N}} = \frac{T_a}{T_{a_N}}\,\frac{\vartheta_N}{\vartheta}$$

and

$$\frac{T_a}{T_{a_N}} = \frac{\theta_N}{\theta}$$

when

$$T_c = T_{c_N}$$

and

$$\frac{1+\frac{G_B}{G_L}}{1+\frac{G_{B_N}}{G_{L_N}}} = \frac{1-\frac{c_p T_{a_N}}{\vartheta_N H_B}}{1-\frac{c_p T_a}{\vartheta_N H_B}} = \frac{1-\frac{c_p T_c}{\theta_N \vartheta_N H_B}}{1-\frac{c_p T_c}{\theta \vartheta(\theta) H_B}}$$

Hence

$$\frac{N_{e_0}}{N_{e_{0_N}}} = \left(\frac{p_a}{p_{a_N}}\right)\left(\frac{\eta_{e_0}}{\eta_{e_{0_N}}}\right)\left(\frac{\vartheta_N - \frac{c_p T_c}{\theta_N H_B}}{\vartheta - \frac{c_p T_c}{\theta H_B}}\right)\sqrt{\frac{\theta_N}{\theta}}\left(\frac{j-1}{j-\frac{\theta}{\theta_N}\left(\frac{\pi_N}{\pi}\right)^{\frac{\eta_L K}{K-1}}}\right) \tag{2.27}$$

or with

$$\frac{p_a}{p_{a_N}} = \left(\frac{T_a}{T_{a_N}}\right)\left(\frac{\gamma_a}{\gamma_{a_N}}\right) = \left(\frac{\theta_N}{\theta}\right)\left(\frac{\gamma_a}{\gamma_{a_N}}\right)$$

$$f = \frac{\frac{N_{e_0}}{\gamma_a}}{\frac{N_{e_{0_N}}}{\gamma_{a_N}}} = \frac{\eta_{e_0}}{\eta_{e_{0_N}}}\left(\sqrt{\frac{\theta_N}{\theta}}\right)\left(\frac{\theta_N \vartheta_N - \frac{c_p T_c}{H_B}}{\theta \vartheta - \frac{c_p T_c}{H_B}}\right)\left(\frac{j-1}{j-\frac{\theta}{\theta_N}\left(\frac{\pi_N}{\pi}\right)^{\frac{\eta_L k}{k-1}}}\right) \tag{2.28}$$

with equations (2.19) and (2.20)

$$
f = \frac{\dfrac{N_{e_0}}{\gamma_a}}{\dfrac{N_{e_{0_N}}}{\gamma_{a_N}}} = \frac{\eta_{e_0}}{\eta_{e_{0_N}}}\sqrt{\frac{\theta_N}{\theta}}\left\{\frac{1 - \dfrac{c_p \Delta T_{B_N}}{H_B}}{\dfrac{\theta_N - \pi_N}{\theta_N - \pi_N\left(\dfrac{\theta}{\theta_N}\right) - \left[1 - \dfrac{1}{\dfrac{\eta_L k}{k-1} - \dfrac{j-1}{h'\left(1-\dfrac{1}{\pi_N}\right)}}\right]} - \dfrac{c_{p_B}\Delta T_{B_N}}{H_B}}\right.
$$

$$
\left.\frac{j-1}{j - \left(\dfrac{\theta}{\theta_N}\right) - \left(\dfrac{h'\dfrac{j-1}{\left(1-\dfrac{1}{\pi_N}\right)}}{\dfrac{\eta_L k}{k-1} - \dfrac{j-1}{h'\left(1-\dfrac{1}{\pi_N}\right)}}\right)}\right\}
$$

(fig. 7)

The right-hand side of this equation depends only on the free variable θ and hence for given T_c depends only on the inlet temperature T_a into the compressor. From the left-hand side of the equation, there is seen the obvious dependence of the output on the air density for the intake into the compressor $\gamma_a = p_a/RT_a$.

The fuel consumption is obtained from

$$g = \frac{G_B/\gamma_a}{G_{B_N}/\gamma_{a_N}} = \sqrt{\frac{\theta_N}{\theta}} \left(\frac{\theta_N \vartheta_N - \dfrac{c_p T_c}{H_B}}{\theta \vartheta - \dfrac{c_p T_c}{H_B}} \right) \left[\frac{j-1}{j - \dfrac{\theta}{\theta} \left(\dfrac{\pi_N}{\pi} \right) \dfrac{\eta_i k}{k-1}} \right] \qquad (2.29a)$$

$$g = \frac{G_B/\gamma_a}{G_{B_N}/\gamma_a} = \sqrt{\frac{\theta_N}{\theta}} \left\{ \frac{\dfrac{1 - \dfrac{c_p \Delta T_{B_N}}{H_B}}{\theta_N - \pi_N}}{\theta_N - \pi_N \left(\dfrac{\theta}{\theta_N} \right)^{-\left[1 - \dfrac{1}{\dfrac{\eta_L k}{k-1} - \dfrac{j-1}{h'\left(1 - \dfrac{1}{\pi_N}\right)}} \right]}} - \frac{c_p \Delta T_{B_N}}{H_B} \right\}$$

$$\frac{j-1}{j - \left(\dfrac{\theta}{\theta_N} \right)^{-\left[\dfrac{h' \dfrac{j-1}{(1-1/\eta_N)}}{\dfrac{\eta_L k}{k-1} - \dfrac{j-1}{h'(1-1/k_N)}} \right]}}$$

whence is obtained the dependence of the fuel consumption on the free variable θ and hence for given T_c on the inlet temperature T_a in the compressor.

Because of the slight variability of the inlet temperature, the assumptions made are admissible. Similarly η_m, η_L, and η_T may be assumed as constant.

For the performance behavior at maximum load for the different flight conditions, there are used in addition to the layout parameters $\pi_N = (T_B/T_a)_N$ and $\theta_N = (T_c/T_a)_N$ the following parameters which characterize the behavior of the compressor and the turbine in deviating from the layout condition:

$$j = \frac{\varphi_{L_N} K_L}{\varphi_{T_N} K_T}$$

and

$$d = \frac{j-1}{h'\left(1-\frac{1}{\pi_N}\right)} = \frac{1}{1-\frac{1}{\pi_N}}\left(\frac{1}{\varphi_{T_N} K_T} - \frac{1}{\varphi_{L_N} K_L}\right)$$

The flight condition itself is suitably characterized by $\theta/\theta_N = T_N/T_a$. Figure 3 shows the effect of h' on the value π/π_N as a function of θ/θ_N. Figure 4 shows for the value of $\theta_N/\pi_N = 1.667$ as an example the effect of h' on the value of ϑ/ϑ_N as a function of θ/θ_N. Figure 5 shows that for $\pi_N = 2$ and $\theta_N = 3.333$ the effect of h' on the internal efficiency and jet efficiency $t_{e_0} = \eta_{e_0}/\eta_{e_{0_N}} = \eta_i/\eta_{i_N}$, as a function of θ/θ_N.

Figure 6 shows the effect of h', for the value of $\theta_N/\pi_N = 1.667$, on the magnitude $g = \frac{G_B/\gamma_a}{G_{B_N}/\gamma_{a_N}}$, which characterizes the fuel consumption as a function of θ/θ_N for: (a) $j = 1.5$; (b) $j = 2.0$; (c) $j = 2.5$; and (d) $j = 3.0$.

Figure 7 shows for $\pi_N = 2$ and $Q_N = 3.333$ as an example the effect of h' on the magnitude $f = \frac{N_{e_0}/\gamma_a}{N_{e_{0_N}}/\gamma_{a_N}} = g\, t_{e_0}$ characterizing the output as a function of Q/Q_N for: (a) $j = 1.5$; (b) $j = 2.0$; (c) $j = 2.5$; and (d) $j = 3.0$. It is seen that the performance behavior as a function of the flight state can be affected in a desirable way at least within limits by the suitable choice of the magnitudes j and k.

[Ed. note: $j = \dfrac{\varphi_{L_N} K_L}{\varphi_{T_N} K_T}$ and $k = \varphi_{L_N} K_L$.]

3. PROPULSIVE AND ECONOMICAL EFFICIENCIES OF JET ENGINE IN FLIGHT FOR REGULATED OUTPUT

With the jet engine in flight, the inlet temperature T_1 at the compressor is greater than the temperature T_0 of the air at rest at the flight altitude h by

$$c_p (T_1 - T_0) = \frac{{v_e}^2}{2g}$$

so that

$$\frac{T_1}{T_0} = 1 + \frac{{v_e}^2}{2gc_pT_0}$$

or with

$$\frac{{v_e}^2}{2gc_pT_0} = \frac{M^2}{5}$$

$$\frac{T_1}{T_0} = 1 + \frac{M^2}{5}$$

By setting

$$\frac{T_0}{\Delta T_B} = \vartheta_0$$

there is obtained with $c_{pL} = c_{pB}$

$$\vartheta = \vartheta_0 \left(1 + \frac{M^2}{5}\right) \tag{3.1}$$

Inasmuch as the thermal propulsive efficiency η_t depends on ϑ, equation (3.1) also gives the dependence of the thermal jet-engine efficiency η_t on the flight velocity.

For flight conditions, it is suitable to characterize the heat loading with the aid of the temperature T_c at the inlet to the turbine, referred to the temperature T_1 at the inlet to the compressor.

But

$$\theta = \frac{T_c}{T_1} = \left(\frac{T_c}{T_{0_N}}\right)\left(\frac{T_{0_N}}{T_0}\right)\left(\frac{T_0}{T_1}\right)$$

Now let

$$\frac{T_c}{T_N} = \theta_N$$

where the previous design condition denotes the condition near the ground. The atmosphere is assumed as the standard. Then

$$\frac{T_N}{T_0} = \frac{1}{1-0.0226\ h} \tag{3.2}$$

(h is altitude in km)

Hence

$$\theta = \frac{\theta_N}{(1-0.0226\ h)\left(1 + \frac{M^2}{5}\right)} \tag{3.3}$$

or setting

$$M_N{}^2 = \frac{v_e{}^2}{w_{s_N}{}^2}$$

[Ed. note: M_N term squared by reviewer.]

where w_{s_N} is the sound velocity at zero altitude.

Then at any flight speed

$$M^2 = M_N{}^2 \left(\frac{w_{s_N}{}^2}{w_s{}^2}\right) = M_N{}^2 \frac{T_N}{T_0} = \frac{M_N{}^2}{1-0.0226\ h}$$

$$\theta = \frac{\theta_N}{(1-0.0226\ h)\left(1 + \frac{M_N{}^2}{5(1-0.0226\ h)}\right)} = \frac{\theta_N}{1-0.0226\ h + \frac{M_N{}^2}{5}} \qquad (3.4)$$

(fig. 8)

With $\theta(h,M_N)$ there is still to be determined the engine efficiency at standstill $\eta_{e_0}(\theta)$ according to equation (2.24) and according to equation (2.23) the internal efficiency $\eta_i(\theta)$.

According to equation (4.23) [Ed. note: No equation 4.23 in paper.], the jet-engine efficiency η_e in flight is greater than the jet-engine efficiency η_{e_0}. With

$$T_0 = T_N\ (1-0.0226\ h)$$

$$M^2 = \frac{M_N{}^2}{1-0.0226\ h}$$

$$\eta_{e_0} \frac{1-\eta_i}{\eta_i} = \eta_d\ (\eta_g-\eta_i)$$

Figure 8 shows the ratio θ/θ_N as a function of the flight condition, that is, on the Mach number, on the flight velocity, and on the surrounding temperature for the corresponding INA flight altitude

$$\vartheta_0 = \frac{T_0}{\Delta T_B} = \left(\frac{T_0}{T_1}\right)\left(\frac{T_1}{\Delta T_B}\right) = \frac{\vartheta(\theta)}{1 + \frac{M^2}{5}} = \frac{\vartheta(\theta)}{1 + \frac{M_N{}^2}{5(1-0.0226\ h)}}$$

$$\eta_e(hM_N) = \eta_{e_0} + \left\{\frac{c_pT_N}{N_B}(1-0.0226\ h) + \left[\eta_d(\eta_g-\eta_1)-(1-\eta_a)\vartheta(\theta)\right]\frac{5}{5+\frac{M_N^2}{1-0.0226\ h}}\right\}$$

$$\frac{\left(\frac{V_e^2}{V_{e_N}}\right)^2 M_N^2}{5(1-0.0226\ h)} \qquad (3.5)$$

The jet-engine efficiency as a function of the altitude and the flight velocity enables also the determination of the propulsive efficiency η_p and the economical efficiency measure η_w of the jet engine. There must also be determined the load coefficient ξ, which by using the relations

$$\vartheta_0 M^2 = \frac{\vartheta(\theta)\ M_N^2}{1-0.0226\ h + \frac{M_N^2}{5}}$$

$$T_0M^2 = T_N\ M_N^2$$

is

$$\xi = \frac{5}{\vartheta_0\ M^2 - \frac{c_pT_0\ M^2}{H_B}} = \frac{5}{\frac{\vartheta(\theta)\ M_N^2}{1-0.0226h + \frac{M_N^2}{5}} - \frac{c_pT_N\ M_N^2}{H_B}}$$

$$\xi\ (hM_N) = \frac{1}{\frac{1}{1-0.0226\ h + \frac{M_N^2}{5}} - \frac{c_pT_N}{\vartheta(\theta)H_B}}\left(\frac{5}{\vartheta(\theta)M_N^2}\right) \qquad (3.6)$$

Furthermore

$$\frac{c_p \, \Delta T_B}{H_B} (hM_N) = \frac{c_p T_0}{\vartheta_0 H_B} = \frac{c_p \, T_N}{\vartheta(\theta) \, H_B}\left(1-0.0226 \, h + \frac{M_N{}^2}{5}\right) \tag{3.7}$$

There is thus obtained the propulsive efficiency as a function of h and η_e

$$\eta_p \, (hM_N) = \frac{2}{\eta_e(hM_N)\xi(hM_N)}\left(\sqrt{\frac{1}{1 - \frac{c_p \Delta T_B}{H_B}(hM_N)} \left[1+\eta_e(hM_N)\xi(h,M_N)\right]} - 1\right) \tag{3.8}$$

and the economical efficiency measure

$$\eta_w(hM_N) = \frac{2}{\xi(hM_N)}\left(\sqrt{\frac{1}{1 - \frac{c_p \Delta T_B}{H_B}(hM_N)} \left[1+\eta_e(hM_N)\xi(hM_N)\right]} - 1\right) \tag{3.9}$$

Figure 9 shows the ratio $f_w = \frac{\eta_w}{\eta_{w_N}}$ for $M = 0.85$, $c_p T_0/H_B = 0.005$ as a function of θ/θ_N for various values of h' and figure 10 the ratio $\vartheta_w{}^2 = f_{-w}g = \frac{\eta_w/p_a}{\eta_{w_N}/p_{a_N}}$ as a function of θ/θ_N for various values of k.

(a) for $j = 1.5$; (b) for $j = 2.0$; (c) for $j = 2.5$; and (d) for $j = 3.0$.

4. THE POWER AVAILABLE IN FLIGHT AT ANY ALTITUDE

The propulsive power N_W available in flight is

$$N_W = \eta_p N_e = \eta_w G_B H_B$$

This power serves to overcome the air resistance of the airplane for climbing and accelerating. If neither climb nor acceleration is required, there corresponds to each flight velocity and each altitude for given weight of the aircraft a horizontal flight power N_H. As long as N_H is smaller than the propulsive power N_W required at any velocity and altitude, the flight can be conducted with corresponding throttling of the engine. The difference between N_H and N_W is available for climb or acceleration.

Because the principal operating condition of a jet engine is essentially that at regulated normal output and because the regulated outputs with a jet engine are of more importance than in flight with the conventional engine where the range of the throttle loads plays a great part, particular consideration will be given here to the propulsive output of the jet engine as a function of the altitude and the speed. It may be remarked that the throttled output can be treated in a similar manner.

The economical efficiency $\eta_w(h,M)$ has been considered. The fuel consumption at regulated load is

$$G_B = G_{B_N} \frac{\gamma_a}{\gamma_{a_N}} g(\theta)$$

where

$$\theta = \frac{\theta_N}{1-0.0226\ h + \frac{M_N^2}{5}}$$

The ratio γ_a/γ_{a_N} of the air densities for zero loss flow utilization is given by

$$\frac{\gamma_a}{\gamma_{a_N}} = \left(\frac{\gamma_1}{\gamma_0}\right)\left(\frac{\gamma_0}{\gamma_{a_N}}\right) = \left(\frac{T_1}{T_0}\right)^{k-1} \frac{\gamma_0}{\gamma_{0_N}}$$

$$\gamma_{a_N} = \gamma_{0_N}$$

$$\gamma_{0_N} = e$$

where $\frac{\gamma_0}{\gamma_{0_N}}$ (h) is obtained from the standard atmosphere. Then approximately

$$\frac{\gamma_0}{\gamma_{0_N}} = e^{-\frac{h}{10}}$$

(where h is in km)
with

$$\frac{T_1}{T_0} = 1 + \frac{M^2}{5} = 1 + \frac{M_N{}^2}{5(1-0.0226\ h)}$$

thus

$$\frac{\gamma_a}{\gamma_{a_N}}\ (hM_N) = \left(1 + \frac{M_N{}^2}{5(1-0.0226\ h)}\right)^{0.4} e^{-\frac{h}{10}}$$

The fuel consumption $G_B(hM_N)$ at maximum load is therefore

$$\frac{G_B}{G_{B_N}} = \frac{\gamma_a}{\gamma_{a_N}}\ (hM_N) = g\left[\theta(hM_N)\right]$$

The propulsive power is obtained from

$$N_w = (hM_N) = \eta_w \, (hM_N) \left[\frac{G_B \, (hM_N)}{G_{B_N}} \right] G_{B_N} \, H_B$$

Both the propulsive normal power N_w and the fuel consumption $N_B = G_B H_B$ decrease essentially with the altitude, corresponding to the air density. This decrease can, however, by suitable design of the compressor and the turbine (by varying h' and j) in relation to each other, be lowered or raised in the desired manner. For this purpose, however, a detailed investigation, which is not gone into here, of these air-flow engines becomes necessary.

5. THE POSSIBILITIES OF OVERLOAD AND THROTTLING

Depending on the characteristics of the compressor and the turbine, there occurs, on increasing the inlet temperature to the turbine, either an increase (case 1) or a decrease (case 2) of the propulsive efficiency and conversely for the lowering of the inlet temperature. In the first case, there is a possibility of at least a short-time overloading if the inlet temperature to the turbine can be increased at least for a short time during this overloading. In the second case, overloading for an arbitrary length of time is possible as long as the compressor does not enter the surging range. This overload will, in general, be associated only with a lowering in the economical efficiency and hence with an uneconomical increase in the fuel consumption.

In the first case, however, a throttling of the engine on lowering the inlet temperature to the turbine for any length of time is entirely possible; in the second case, on account of the increase in the inlet temperature, it is possible only for a short time. This type of throttling is generally less economical, however, than throttling by lowering the rotational speed, because with the lowering in the rotational speed the efficiency of the compressor should, in general, rise somewhat.

6. SUMMARY

The rotational speed and the quantity of air flow and therefore the compressor pressure and inlet temperature can be regulated by the fuel supply and the adjustment of the outlet nozzle. The regulation for constant Mach number at the inlet to the compressor and

for constant inlet temperature to the turbine was investigated. The normal output thereby obtained was determined by the characteristic curves of the compressor and the turbine in relation to each other. The propulsive efficiency and economy in flight for normal output and the normal output available at any flight altitude are discussed and the possibility of overloading and throttling is considered.

Translated by S. Reiss,
National Advisory Committee
for Aeronautics.

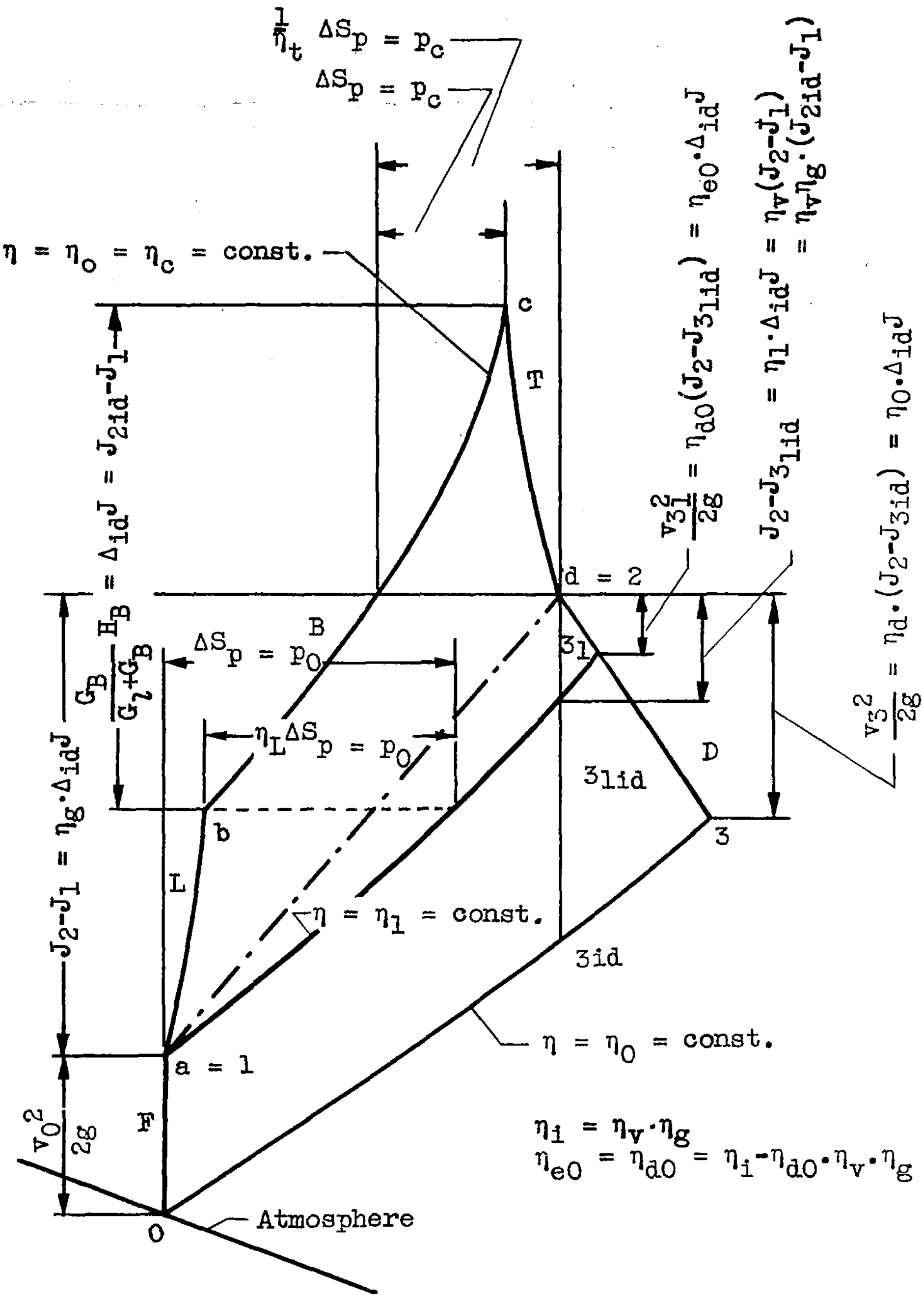

Figure 1. - Entropy-enthalpy diagram for explaining mode of operation of jet engine and effect of flight speed.

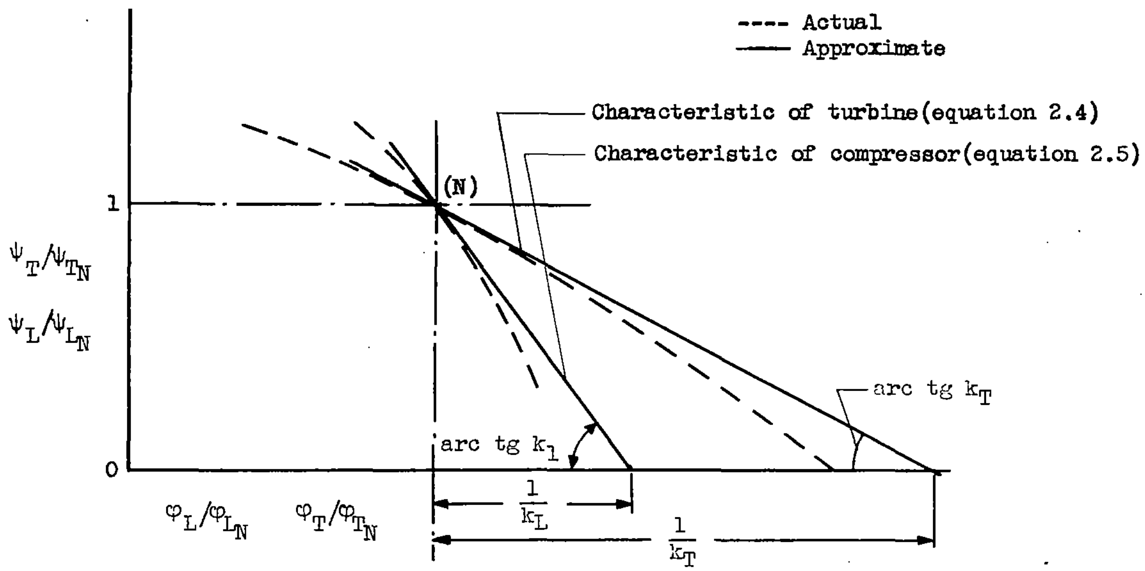

Figure 2. - Characteristics of compressor L and turbine T of jet engine normalized characteristic to design state N.

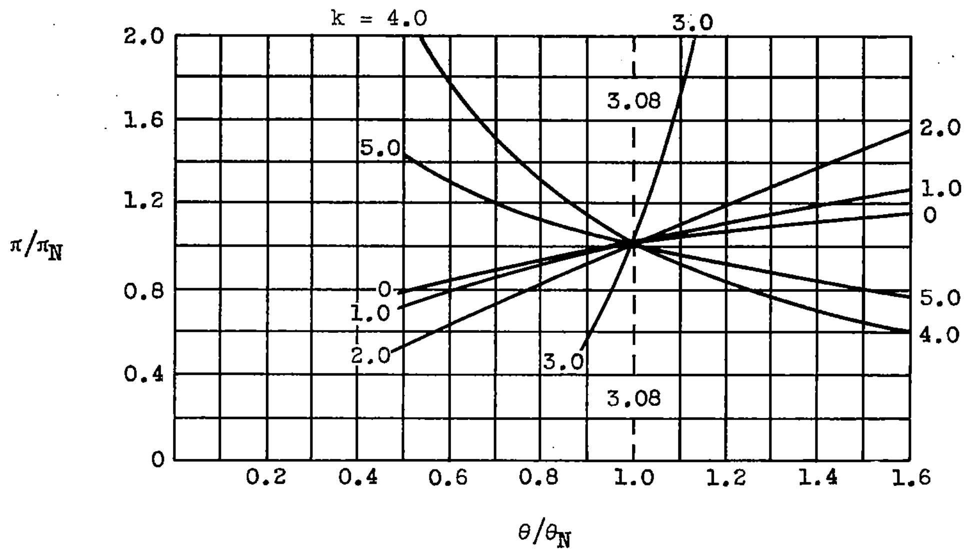

Figure 3. - Dependence of characteristic parameters π/π_N and θ/θ_N on each other for various values of design constant k.

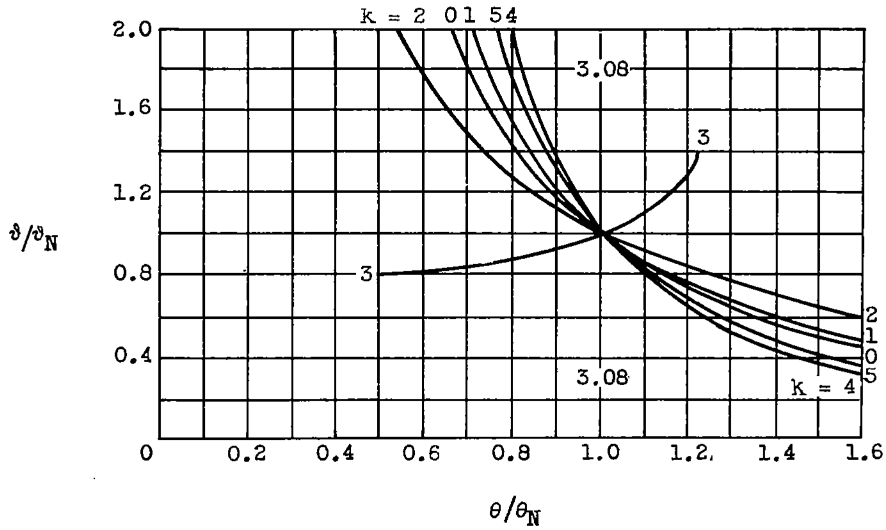

Figure 4. - Dependence of the characteristic parameters ϑ/ϑ_N and θ/θ_N on each other for various values of design constant k.

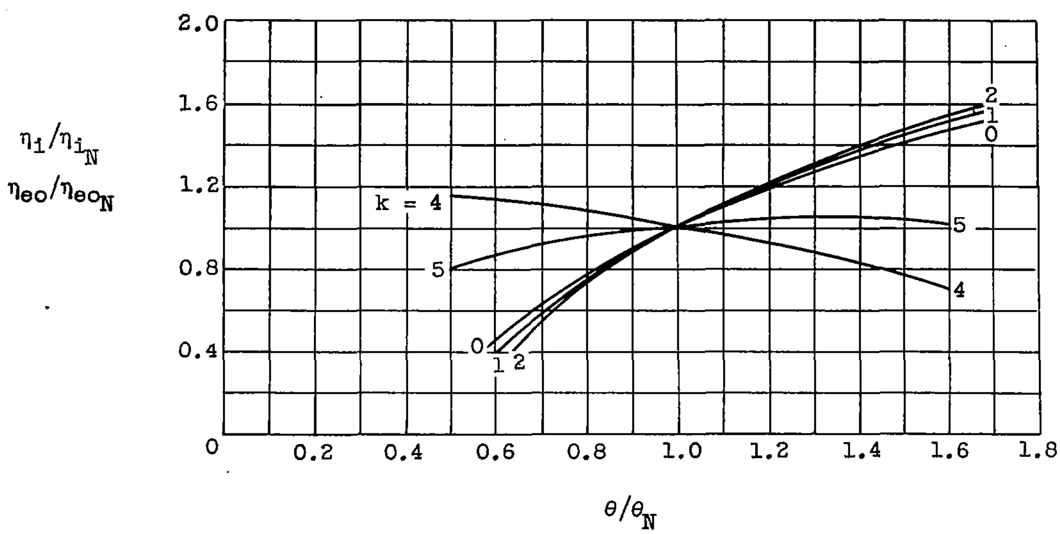

Figure 5. - Dependence of internal and useful efficiencies on characteristic parameter θ/θ_N for various values of design constant k.

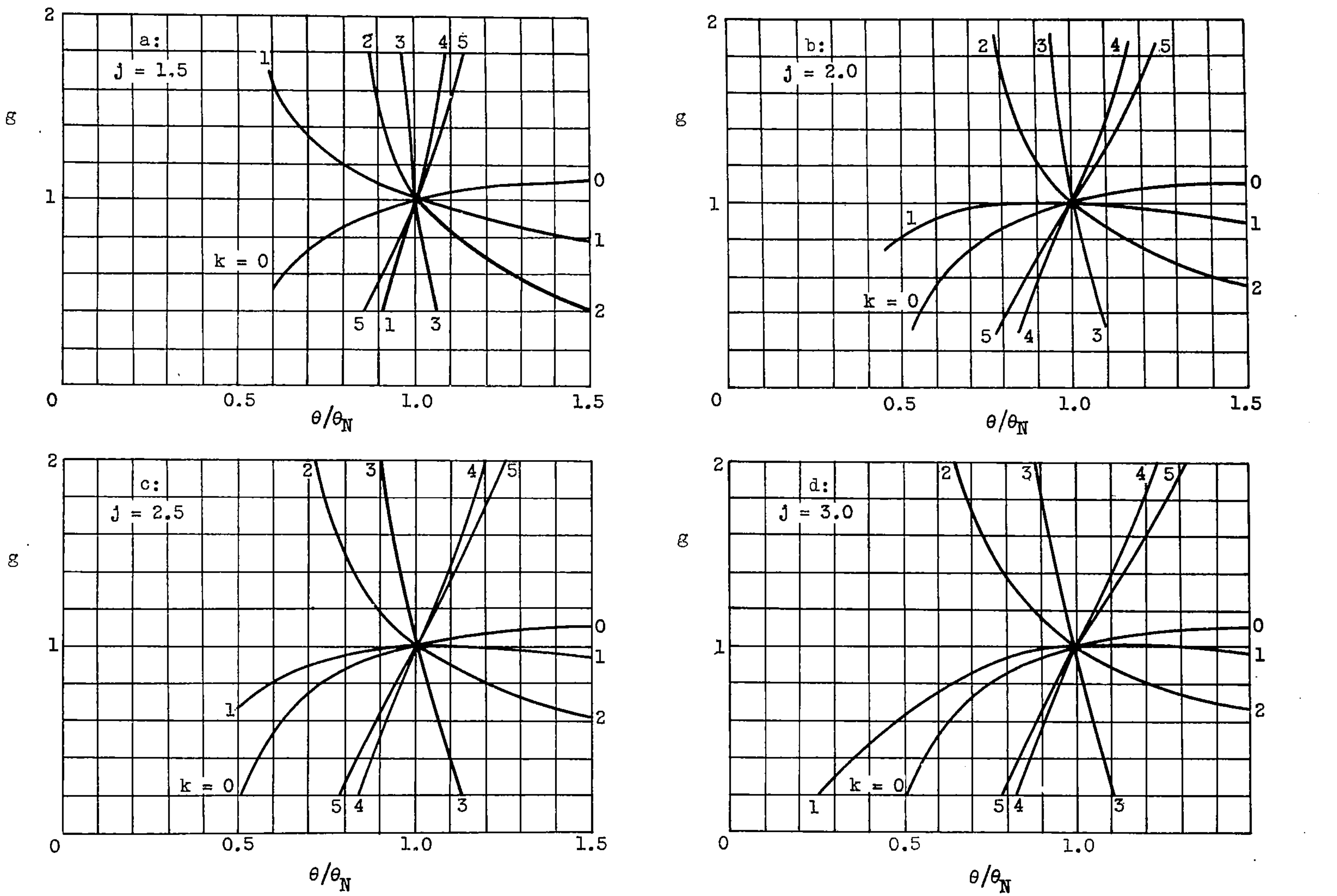

Figure 6. - Dependence of ratio g of fuel consumption on characteristic parameter θ/θ_N for various values of parameter k and j.

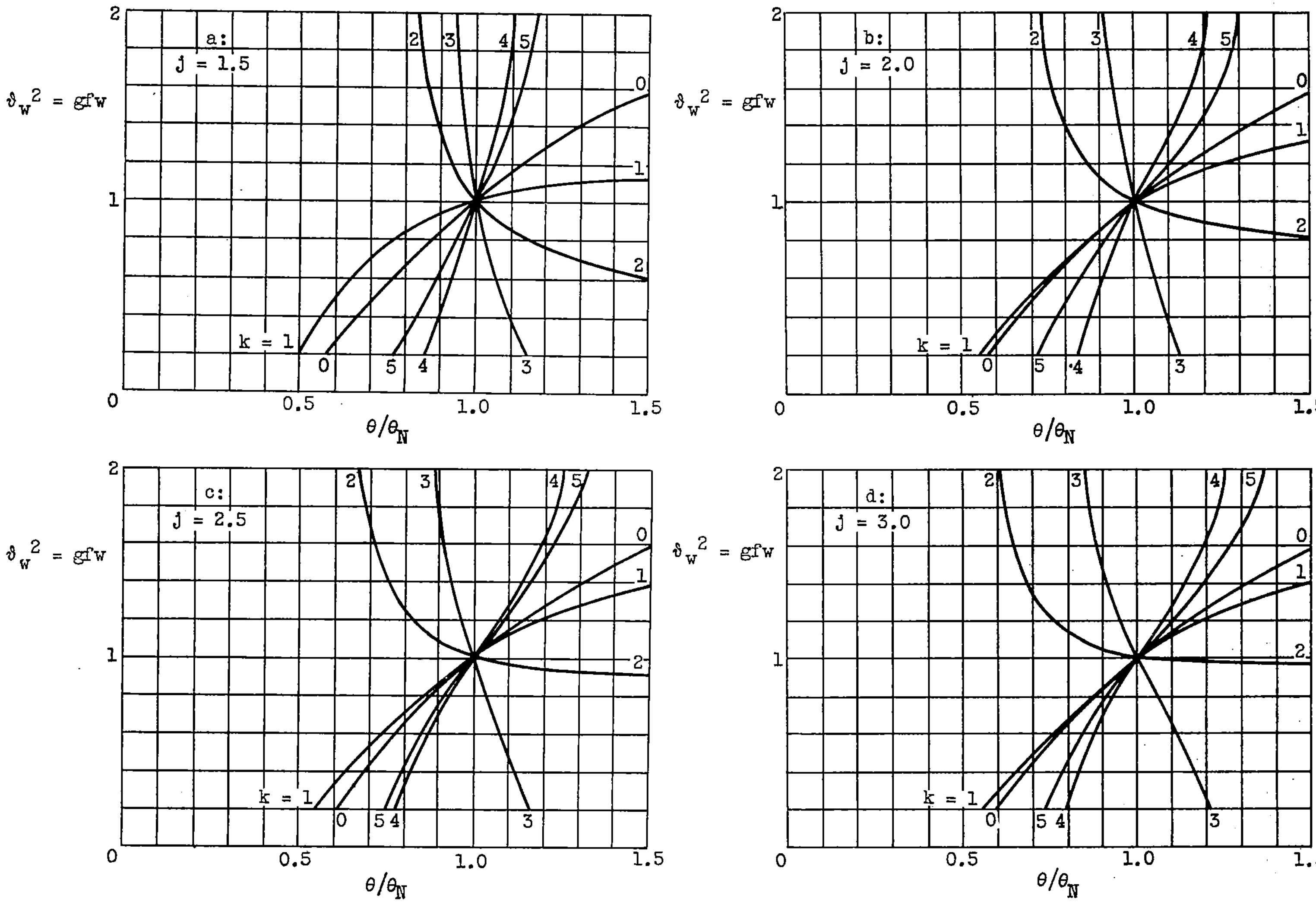

Figure 7. - Dependence of ratio P_{eo} of output for various operating conditions (θ/θ_N) for different values of design constant k and the magnitude j.

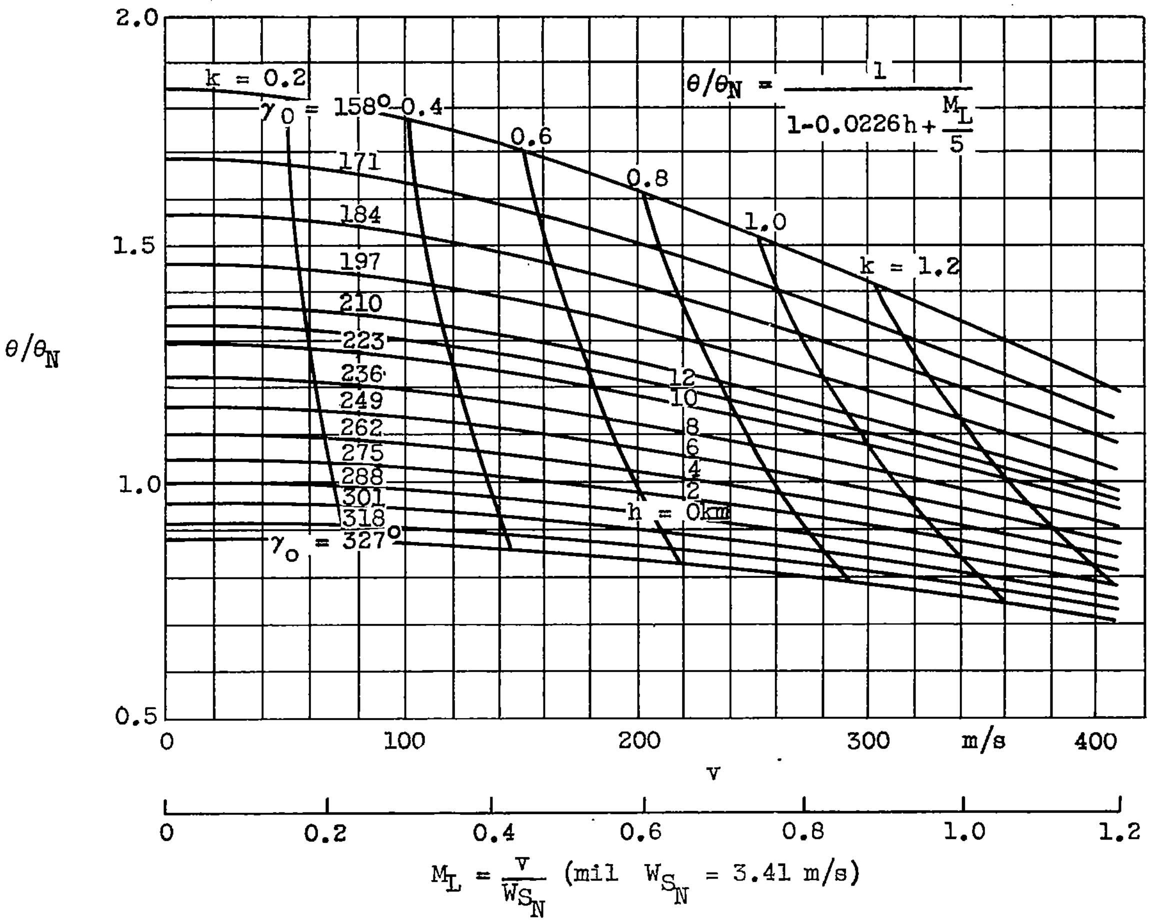

Figure 8. - Dependence of characteristic parameter θ/θ_N on flight altitude h and flight velocity v_e for constant Mach number M_L of compressor and for constant inlet temperature to turbine.

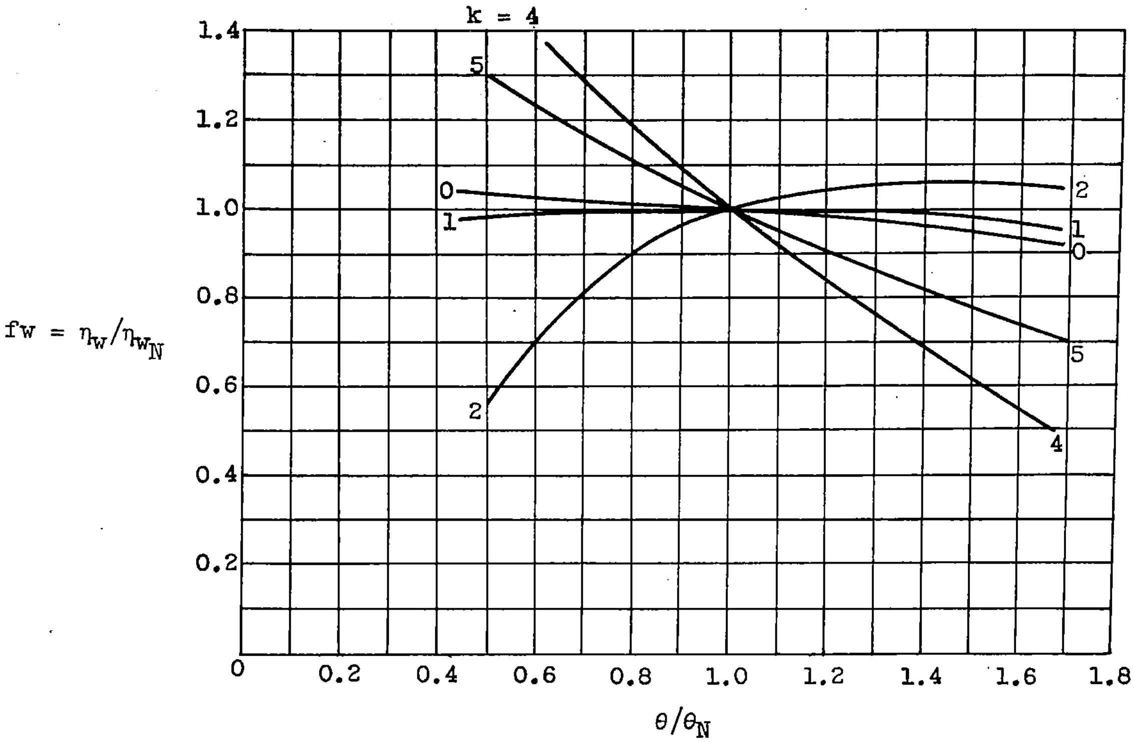

Figure 9. - Dependence of economical efficiency on different values of design constant k for flight Mach number M = 0.85 and for $\eta_d = 0.97$, $\eta_m = 0.94$, $\eta_T = 0.91$, $\eta_L = 0.68$. ($c_pT_0/H_B = 0.005$)

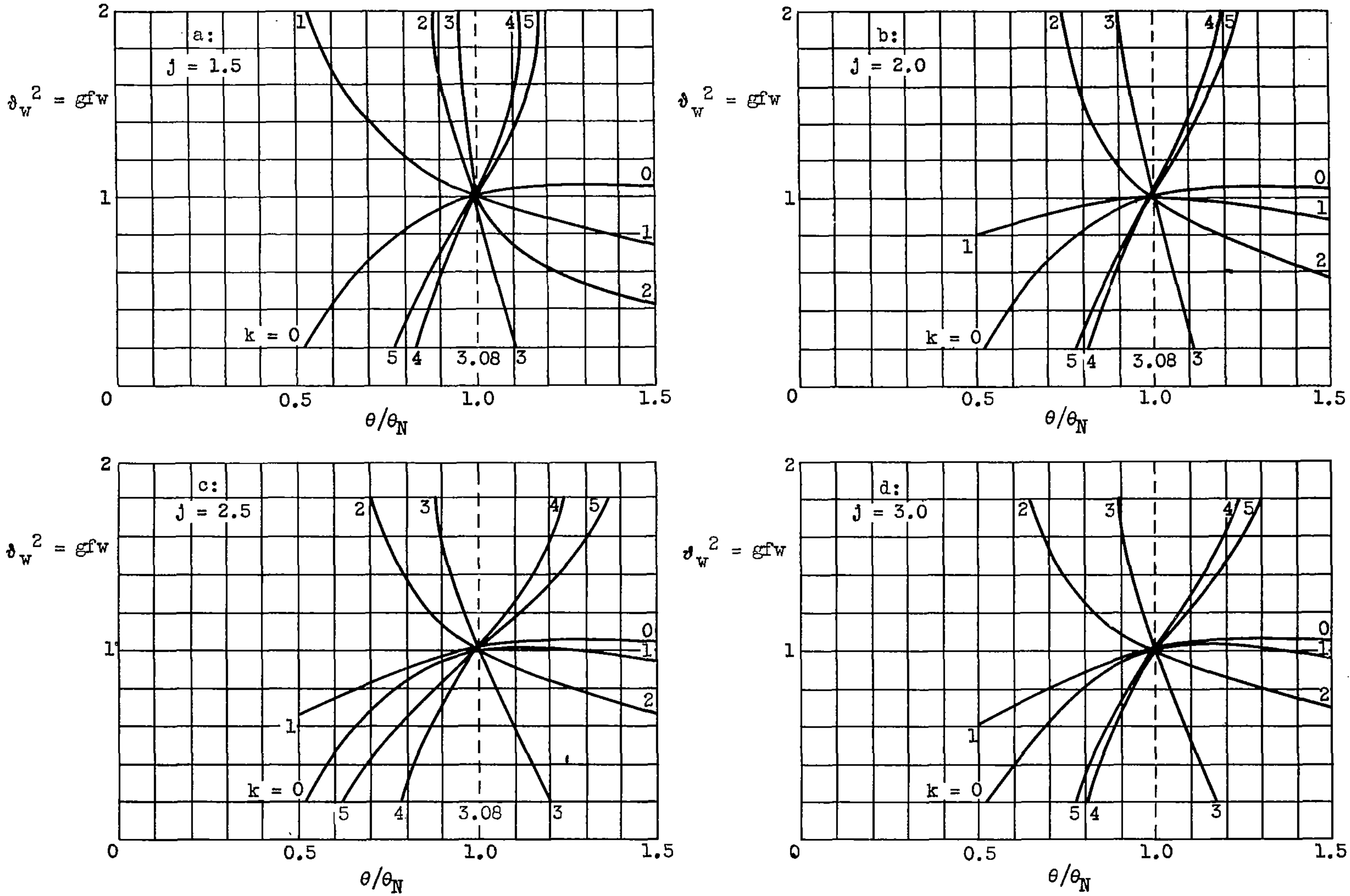

Figure 10. - Dependence of propulsive load on operating condition for various values of design constant k and magnitude j for flight Mach number M = 0.85 and for $\eta_d = 0.97$, $\eta_m = 0.94$, $\eta_T = 0.91$, $\eta_L = 0.68$. ($c_pT_o/H_B = 0.005$).

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